Exploring CP Violation and Penguin Effects through $B_d^0 \to D^+ D^-$ and $B_s^0 \to D_s^+ D_s^-$

Robert Fleischer

Theory Division, Department of Physics, CERN, CH-1211 Geneva 23, Switzerland

Abstract

The decay $B_d^0 \to D^+ D^-$ offers an interesting probe of CP violation, but requires control of penguin effects, which can be done through $B_s^0 \to D_s^+ D_s^-$ by means of the U-spin flavour symmetry of strong interactions. Recently, the Belle collaboration reported indications of large CP violation in the B_d^0 decay, which were, however, not confirmed by BaBar, and first signals of the B_s^0 channel were observed at the Tevatron. In view of these developments and the quickly approaching start of the LHC, we explore the allowed region in observable space for CP violation in $B_d^0 \to D^+ D^-$, perform theoretical estimates of the relevant hadronic penguin parameters and observables, and address questions both about the most promising strategies for the extraction of CP-violating phases and about the interplay with other measurements of CP violation and the search for new physics. As far as the latter aspect is concerned, we point out that the $B_q^0 \to D_q^+ D_q^-$ system provides a setting for the determination of the B_q^0–$\bar{B}_q^0$ mixing phases ($q \in \{d, s\}$) that is complementary to the conventional $B_d^0 \to J/\psi K_S$ and $B_s^0 \to J/\psi \phi$ modes with respect to possible new-physics effects in the electroweak penguin sector.
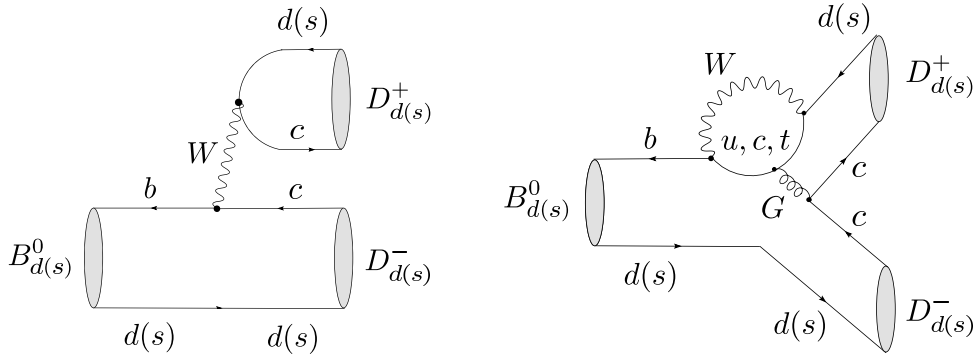
Figure 1: Tree and penguin topologies contributing to the U-spin-related $B_d^0 \to D^+ D^-$ and $B_s^0 \to D_s^+ D_s^-$ decays.

1   Introduction

Decays of B mesons are subject of intensive investigations. Thanks to the interplay between a lot of theoretical work and the data from the $e^+ e^-$ B factories with their detectors BaBar (SLAC) and Belle (KEK) as well as from the Tevatron (FNAL), valuable insights into CP violation could be obtained during the recent years. In the Standard Model (SM), this phenomenon is closely related to the Cabibbo–Kobayashi–Maskawa (CKM) matrix [1], and can be characterized by the unitarity triangle (UT) with its three angles α, β and γ. This adventure will soon be continued at the LHC (CERN), with its dedicated B-decay experiment LHCb.

In the B-physics landscape, an interesting probe of CP violation is also offered by $B_d^0 \to D^+ D^-$. As can be seen in Fig. 1, this decay originates from $\bar{b} \to \bar{c} c \bar{d}$ quark-level processes, and receives contributions both from a colour-allowed tree-diagram-like topology and from penguin diagrams. In analogy to the prominent $B_d^0 \to \pi^+ \pi^-$ decay, the latter contributions lead to complications of the theoretical interpretation of the CP-violating observables. However, the penguin effects can fortunately be controlled by means of the $B_s^0 \to D_s^+ D_s^-$ channel [2], which is related to $B_d^0 \to D^+ D^-$ through an interchange of all down and strange quarks, as can also be seen in Fig. 1. Because of this feature, the U-spin flavour symmetry of strong interactions allows us to derive relations between non-perturbative hadronic parameters,[1] so that the measurement of CP violation in $B_d^0 \to D^+ D^-$ can be converted into CP-violating weak phases. In comparison with conventional flavour-symmetry strategies [3], the advantage of the U-spin method is that no additional dynamical assumptions are needed, and that also electroweak (EW) penguin contributions are automatically included.

The key observables are the CP-averaged branching ratios as well as the direct and mixing-induced CP asymmetries $\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \to D^+ D^-)$ and $\mathcal{A}_{\text{CP}}^{\text{mix}}(B_d \to D^+ D^-)$, respectively, which enter the following time-dependent rate asymmetry [4]:

$$\mathcal{A}_{\text{CP}}(B_d(t) \to D^+ D^-) \equiv \frac{\Gamma(B_d^0(t) \to D^+ D^-) - \Gamma(\bar{B}_d^0(t) \to D^+ D^-)}{\Gamma(B_d^0(t) \to D^+ D^-) + \Gamma(\bar{B}_d^0(t) \to D^+ D^-)}$$

---

[1]The U-spin flavour symmetry connects strange and down quarks in the same way through $SU(2)$ transformations as the isospin symmetry connects the up and down quarks.

$$= \mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to D^+D^-) \cos(\Delta M_d t) + \mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to D^+D^-) \sin(\Delta M_d t), \qquad (1)$$

where ΔM_d is the mass difference of the B_d mass eigenstates. The Belle collaboration has recently reported evidence for CP violation in $B_d^0 \to D^+D^-$, which could not be confirmed by BaBar. The current status reads as follows:

$$\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to D^+D^-) = \begin{cases} +0.11 \pm 0.22 \pm 0.07 \ (\text{BaBar [5]}) \\ -0.91 \pm 0.23 \pm 0.06 \ (\text{Belle [6]}) \end{cases} \qquad (2)$$

$$\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to D^+D^-) = \begin{cases} +0.54 \pm 0.34 \pm 0.06 \ (\text{BaBar [5]}) \\ +1.13 \pm 0.37 \pm 0.09 \ (\text{Belle [6]}); \end{cases} \qquad (3)$$

the Heavy Flavour Averaging Group (HFAG) gives the following averages [7]:

$$\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to D^+D^-) = -0.37 \pm 0.17, \quad \mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to D^+D^-) = 0.75 \pm 0.26, \qquad (4)$$

which have to be taken with great care in view of the inconsistency between the BaBar and Belle measurements. Concerning the CP-averaged branching ratio, we have

$$\mathrm{BR}(B_d \to D_d^+D_d^-) = \begin{cases} (2.8 \pm 0.4 \pm 0.5) \times 10^{-4} & (\text{BaBar [8]}) \\ (1.97 \pm 0.20 \pm 0.20) \times 10^{-4} & (\text{Belle [6]}), \end{cases} \qquad (5)$$

yielding the average of $\mathrm{BR}(B_d \to D_d^+D_d^-) = (2.11 \pm 0.26) \times 10^{-4}$. Thanks to the updated Belle result, this number is now about $1.6\,\sigma$ lower than the HFAG value of $\mathrm{BR}(B_d \to D_d^+D_d^-) = (3.0 \pm 0.5) \times 10^{-4}$ [7]. The CDF collaboration has recently observed the first signals of the $B_s^0 \to D_s^+D_s^-$ decay [9], which correspond to the CP-averaged branching ratio

$$\mathrm{BR}(B_s \to D_s^+D_s^-) = (1.09 \pm 0.27 \pm 0.47)\%. \qquad (6)$$

Performing a run on the $\Upsilon(5S)$ resonance, also the Belle collaboration has recently obtained an upper bound of 6.7% (90% C.L.) for this branching ratio [10]. Moreover, the D0 collaboration has performed a first analysis of the combined $B_s \to D_s^{(*)}D_s^{(*)}$ branching ratio, with the result of $\mathrm{BR}(B_s \to D_s^{(*)}D_s^{(*)}) = (3.9^{+1.9+1.6}_{-1.7-1.5})\%$ [11].

Although the current experimental picture is still in an early stage, it raises several questions, which are further motivated by the quickly approaching start of the LHC:

- What is the allowed SM region for the CP violation in $B_d^0 \to D^+D^-$?

- What are the most promising strategies for the extraction of weak phases?

- What is the interplay with other measurements of CP violation and the search for new physics (NP)?

These items are the central target of this paper. It is organized as follows: in Section 2, we explore the parameter space of the CP-violating $B_d^0 \to D^+D^-$ asymmetries, taking also the constraints from $B_s^0 \to D_s^+D_s^-$ and similar modes into account, and perform a theoretical estimate of the corresponding observables in Section 3. In Section 4, we discuss the extraction of CP-violating phases from the $B_d^0 \to D^+D^-$ and $B_s^0 \to D_s^+D_s^-$ decays, while the interplay with other CP probes is discussed in Section 5. Finally, we summarize our conclusions in Section 6.
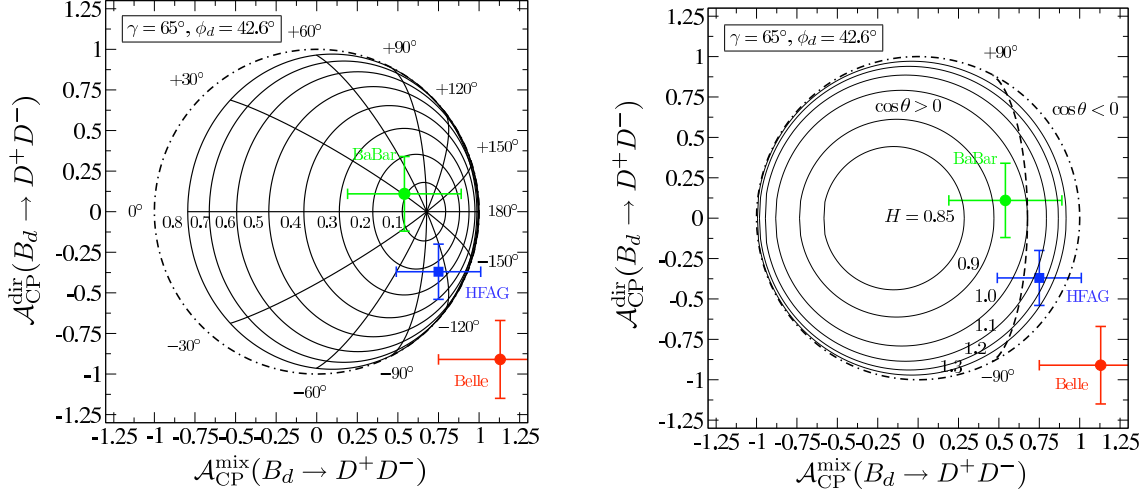
Figure 2: The situation in the $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to D^+D^-)$–$\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to D^+D^-)$ plane. Left panel: contours following from the general SM parametrization; right panel: constraints following from a measurement of the quantity H.

2 CP Violation in $B_d^0 \to D^+D^-$

2.1 Standard Model Expressions

In the SM, we may write the $B_d^0 \to D^+D^-$ decay amplitude as follows [2]:

$$A(B_d^0 \to D^+D^-) = -\lambda \mathcal{A} \left[1 - ae^{i\theta}e^{i\gamma} \right], \tag{7}$$

where λ is the well-known Wolfenstein parameter of the CKM matrix [12], $\mathcal{A}$ denotes a CP-conserving strong amplitude that is governed by the tree contributions, while the CP-consering hadronic parameter $ae^{i\theta}$ measures – sloppily speaking – the ratio of penguin to tree amplitudes. Applying the well-known formalism to calculate the CP-violating observables that are provided by the time-dependent rate asymmetry in (1), we obtain the following expressions:

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to D^+D^-) = \frac{2a\sin\theta\sin\gamma}{1 - 2a\cos\theta\cos\gamma + a^2} \tag{8}$$

$$\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to D^+D^-) = \frac{\sin\phi_d - 2a\cos\theta\sin(\phi_d + \gamma) + a^2\sin(\phi_d + 2\gamma)}{1 - 2a\cos\theta\cos\gamma + a^2}, \tag{9}$$

where ϕ_d denotes the B_d^0–$\bar{B}_d^0$ mixing phase, which takes the value of 2β in the SM. This quantity has been measured at the B factories with the help of the "golden" decay $B_d^0 \to J/\psi K_{\rm S}$ and similar modes, including $B_d \to J/\psi K^*$ and $B_d \to D^*D^*K_{\rm S}$ channels to resolve a twofold ambiguity, as follows [7]:

$$\phi_d = (42.6 \pm 2)^\circ. \tag{10}$$

Concerning the angle γ, the SM fits of the UT obtained by the UTfit and CKMfitter collaborations [13, 14] yield $\gamma = (64.6 \pm 4.2)^\circ$ and $\gamma = (59.0^{+9.2}_{-3.7})^\circ$, respectively. A

3

recent analysis of the U-spin-related $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ transitions finds $\gamma = (66.6^{+4.3+4.0}_{-5.0-3.0})^\circ$ [15], in excellent agreement with these fits. A similar picture emerges also from other recent γ determinations from $B \to \pi\pi, \pi K$ decays [16,17]. Thanks to the LHCb experiment [18], our knowledge of γ will soon improve dramatically, also since very accurate "reference" determinations through pure tree decays will become available. In the limit of $a \to 0$, expression (9) would allow a straighforward extraction of $\sin\phi_d$. However, these penguin effects cannot simply be neglected and require further work.

For the following discussion, we shall assume $\gamma = 65^\circ$ and $\phi_d = 42.6^\circ$. Using (8) and (9), we can then calculate contours in the $\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to D^+D^-)$–$\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to D^+D^-)$ plane for given values of a and θ, which are theoretically exact in the SM. The resulting picture is shown in the left panel of Fig. 2 (for its $B_d^0 \to \pi^+\pi^-$ counterpart, see Ref. [19]). There we have also included the experimental BaBar and Belle results, as well as the HFAG average; the dot-dashed circle defines the outer boundary in this observable space that follows from the general relation

$$\left[\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to D^+D^-)\right]^2 + \left[\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to D^+D^-)\right]^2 \leq 1. \tag{11}$$

Fig. 2 shows that the Belle result lies outside of the physical region, in contrast to the BaBar measurement and the HFAG average. The contours of that figure allow us to read off the corresponding values of a and θ straightforwardly.

2.2 Constraints from $B_s^0 \to D_s^+ D_s^-$

We now go one step further by using the information that is offered by the $B_s^0 \to D_s^+ D_s^-$ decay. In analogy to (7), its SM amplitude can be written as follows:

$$A(B_s^0 \to D_s^+ D_s^-) = \left(1 - \frac{\lambda^2}{2}\right)\mathcal{A}'\left[1 + \epsilon a' e^{i\theta'} e^{i\gamma}\right], \tag{12}$$

where

$$\epsilon \equiv \frac{\lambda^2}{1 - \lambda^2} = 0.05. \tag{13}$$

Following Ref. [2], we introduce

$$H \equiv \frac{1}{\epsilon}\left|\frac{\mathcal{A}'}{\mathcal{A}}\right|^2 \left[\frac{M_{B_d}}{M_{B_s}}\frac{\Phi(M_{D_s}/M_{B_s}, M_{D_s}/M_{B_s})}{\Phi(M_{D_d}/M_{B_d}, M_{D_d}/M_{B_d})}\frac{\tau_{B_s}}{\tau_{B_d}}\right]\left[\frac{\mathrm{BR}(B_d \to D^+D^-)}{\mathrm{BR}(B_s \to D_s^+ D_s^-)}\right]$$

$$= \frac{1 - 2a\cos\theta\cos\gamma + a^2}{1 + 2\epsilon a'\cos\theta'\cos\gamma + \epsilon^2 a'^2}, \tag{14}$$

where

$$\Phi(x,y) \equiv \sqrt{[1 - (x+y)^2][1 - (x-y)^2]} \tag{15}$$

is the well-known $B \to PP$ phase-space function, and the $\tau_{B_{d,s}}$ are the $B_{d,s}$ lifetimes. Applying the U-spin flavour symmetry, we obtain the relations

$$a' = a, \quad \theta' = \theta. \tag{16}$$

4

Thanks to the ϵ suppression in (14), the impact of U-spin-breaking corrections to (16) is marginal for H. In the case of $|\mathcal{A}'/\mathcal{A}|$, ratios of U-spin-breaking decay constants and form factors enter. If we apply the "factorization" approximation, we obtain

$$\left|\frac{\mathcal{A}'}{\mathcal{A}}\right|_{\text{fact}} = \frac{(M_{B_s} - M_{D_s})\sqrt{M_{B_s}M_{D_s}}\,(w_s+1)\,f_{D_s}\,\xi_s(w_s)}{(M_{B_d} - M_{D_d})\sqrt{M_{B_d}M_{D_d}}\,(w_d+1)\,f_{D_d}\,\xi_d(w_d)}, \tag{17}$$

where the restrictions form the heavy-quark effective theory for the $B_q \to D_q$ form factors have been taken into account by introducing appropriate Isgur–Wise functions $\xi_q(w_q)$ with $w_q = M_{B_q}/(2M_{D_q})$ [20]. Studies of the light-quark dependence of the Isgur–Wise function were performed within heavy-meson chiral perturbation theory, indicating an enhancement of ξ_s/ξ_d at the level of 5% [21]. Applying the same formalism to f_{D_s}/f_{D_d} gives values at the 1.2 level [22], which is in accordance with the recent measurement by the CLEO collaboration [23]:

$$\frac{f_{D_s}}{f_{D_d}} = 1.23 \pm 0.11 \pm 0.04, \tag{18}$$

as well as with lattice QCD calculations, as summarized in Ref. [24]. Using heavy-meson chiral perturbation theory and the $1/N_C$ expansion, non-factorizable $SU(3)$-breaking corrections were found at the level of a few percent in Ref. [25]. The CDF result in (6) and the average of (5) yield then, with the CLEO measurement in (18), the following numbers:

$$H = 0.59 \pm 0.31 \quad (0.84 \pm 0.45), \tag{19}$$

where we have added the errors in quadrature, and have also given the result corresponding to the HFAG value of $\text{BR}(B_d \to D^+D^-)$ in parentheses. The general expression for H in (14) implies a lower bound [26], which is given by

$$H \geq \left[1 - 2\epsilon\cos^2\gamma + \mathcal{O}(\epsilon^2)\right]\sin^2\gamma \overset{\gamma=65°}{\longrightarrow} 0.81. \tag{20}$$

Consequently, the rather low central value of (19), which is essentially due to the new Belle result [6], is disfavoured by the experimental information on γ.

If we replace the s spectator quark of the $B_s^0 \to D_s^+D_s^-$ decay through a d quark, we obtain the $B_d^0 \to D_s^+D^-$ process. Whereas the $B_{d(s)} \to D_{d(s)}^+D_{d(s)}^-$ system receives contributions from tree and penguin as well as exchange (E) and penguin annihilation (PA) topologies (the latter are not shown in Fig. 1), the $B_d^0 \to D_s^+D^-$ channel and its U-spin partner $B_s^0 \to D^+D_s^-$ receive only tree and penguin contributions. Consequently, if we use the $SU(3)$ flavour symmetry and assume that the exchange and penguin annihilation topologies play a minor rôle, we may replace $B_s^0 \to D_s^+D_s^-$ in the determination of H through $B_d^0 \to D_s^+D^-$ [27].[2] Expression (14) is then modified as follows:

$$H \approx \frac{1}{\epsilon}\left(\frac{f_{D_s}}{f_{D_d}}\right)^2\left[\frac{\Phi(M_{D_s}/M_{B_d}, M_{D_d}/M_{B_d})}{\Phi(M_{D_d}/M_{B_d}, M_{D_d}/M_{B_d})}\right]\left[\frac{\text{BR}(B_d \to D^+D^-)}{\text{BR}(B_d \to D_s^{\pm}D^{\mp})}\right]. \tag{21}$$

[2]This is analogous to the replacement of $B_s^0 \to K^+K^-$ through $B_d^0 \to \pi^-K^+$ [28].

The importance of the $E+PA$ amplitude can actually be probed through the U-spin related $B_{d(s)} \to D^+_{s(d)}D^-_{s(d)}$ decays. The current experimental situation can be summarized as follows:

$$\text{BR}(B_d \to D^\pm_s D^\mp) = \begin{cases} (6.4 \pm 1.3 \pm 1.0) \times 10^{-3} & (\text{BaBar [29]}) \\ (7.5 \pm 0.2 \pm 0.8 \pm 0.8) \times 10^{-3} & (\text{Belle [30]}), \end{cases} \qquad (22)$$

yielding the average of $\text{BR}(B_d \to D^\pm_s D^\mp) = (7.1 \pm 0.9) \times 10^{-3}$; Belle reported also the upper limit of $\text{BR}(B_d \to D^+_s D^-_s) < 3.6 \times 10^{-5}$ (90% C.L.) [30]. Expression (21) gives then

$$H = 0.85 \pm 0.19 \quad (1.22 \pm 0.31), \qquad (23)$$

where the notation is as in (19).

Let us now investigate the constraints on the $\mathcal{A}^{\text{mix}}_{\text{CP}}(B_d \to D^+D^-)$–$\mathcal{A}^{\text{dir}}_{\text{CP}}(B_d \to D^+D^-)$ plane that follow from H. If we use (14) with (16), we may eliminate the strong phase θ in (8) and (9) with the help of

$$\cos\theta = \frac{1 - H + (1 - \epsilon^2 H)a^2}{2a(1 + \epsilon H)\cos\gamma}, \quad \sin\theta = \pm\sqrt{1 - \cos^2\theta}. \qquad (24)$$

If we then keep a as a free parameter, we arrive at the situation shown in the right panel of Fig. 2, where the dashed line separates the regions with $\cos\theta > 0$ and $\cos\theta < 0$. In the factorization approximation, we expect a negative value of $\cos\theta$. Although non-factorizable effects could generate a large value of θ, we do not expect that $\cos\theta$ changes its sign. This feature is in fact observed for other non-leptonic B-meson decays, such as the $B^0_d \to \pi^+\pi^-$, $B^0_d \to \pi^-K^+$ system [15]. With $\gamma \sim 65°$, which corresponds to $\cos\gamma > 0$, the expression in (14) implies then $H > 1$. In the right panel of Fig. 2, this leaves us with the banana-shaped region in the $\mathcal{A}^{\text{mix}}_{\text{CP}}(B_d \to D^+D^-)$–$\mathcal{A}^{\text{dir}}_{\text{CP}}(B_d \to D^+D^-)$ plane. Interestingly, the central value of the HFAG average falls well into this region, whereas the central value of the BaBar result would require a positive value of $\cos\theta$. Although the current errors are too large to draw definite conclusions, this exercise illustrates the usefulness of the plots in observable space to monitor the experimental picture. Since the B_s input for the determination of H is just the CP-averaged $B_s \to D^+_s D^-_s$ branching ratio, this measurement would also be interesting for an e^+e^- (super-)B factory operating at the $\Upsilon(5S)$ resonance [10, 31].

3 Theoretical Estimates

In order to analyze the $B^0_d \to D^+D^-$ decay theoretically, we "integrate out" the heavy degrees of freedom, i.e. the W boson and top quark in Fig. 1, and use an appropriate low-energy effective Hamiltonian, which takes the following form [32]:

$$\mathcal{H}_{\text{eff}} = \frac{G_{\text{F}}}{\sqrt{2}} \left[\lambda^{(d)}_u \sum_{k=1}^{2} C_k(\mu)Q^{ud}_k + \lambda^{(d)}_c \sum_{k=1}^{2} C_k(\mu)Q^{cd}_k - \lambda^{(d)}_t \sum_{k=3}^{10} C_k(\mu)Q^d_k \right]. \qquad (25)$$

Here the $\lambda^{(d)}_j \equiv V_{jd}V^*_{jb}$ denote CKM factors, Q^{jd}_1 and Q^{jd}_2 ($j \in \{u, c\}$) are the usual current–current operators, $Q^d_3, \ldots, Q^d_6$ and $Q^d_7, \ldots, Q^d_{10}$ denote the QCD and EW penguin
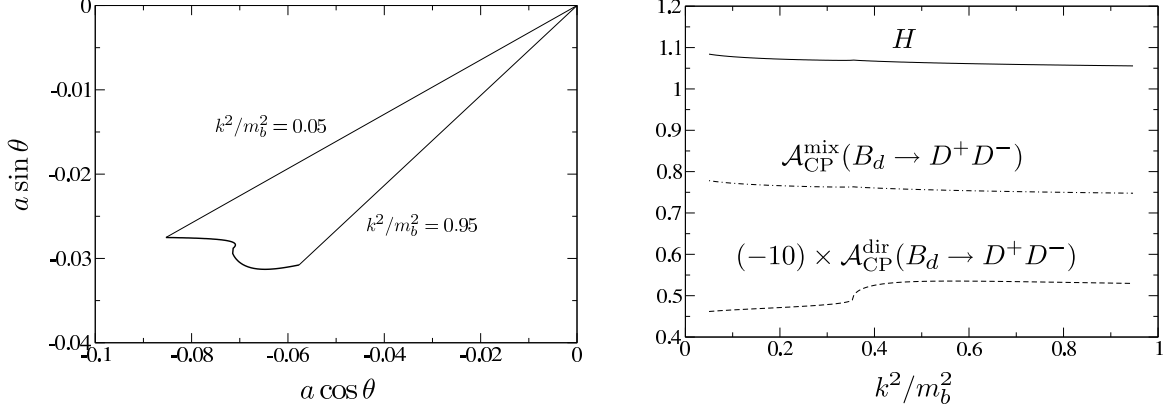
Figure 3: Theoretical estimates of the hadronic parameter $ae^{i\theta}$ (left panel), and the $B_{d(s)} \to D_{d(s)}^+ D_{d(s)}^-$ observables (right panel) for $\gamma = 65°$, $\phi_d = 42.6°$ and $R_b = 0.45$.

operators, respectively, and $\mu = \mathcal{O}(m_b)$ is a renormalization scale. If we apply the Bander–Silverman–Soni mechanism [33] as well as the formalism developed in Ref. [34], we obtain the following estimate:

$$ae^{i\theta} \approx R_b \left[\frac{\mathcal{A}_t + \mathcal{A}_u}{\mathcal{A}_{\mathrm{T}} + \mathcal{A}_t + \mathcal{A}_c} \right], \tag{26}$$

where $R_b \propto |V_{ub}/V_{cb}|$ is the corresponding side of the UT, and

$$\mathcal{A}_{\mathrm{T}} = \frac{1}{3} \overline{C}_1 + \overline{C}_2 \tag{27}$$

$$\mathcal{A}_t = \frac{1}{3} \left[\overline{C}_3 + \overline{C}_9 + \chi_D \left(\overline{C}_5 + \overline{C}_7 \right) \right] + \overline{C}_4 + \overline{C}_{10} + \chi_D \left(\overline{C}_6 + \overline{C}_8 \right) \tag{28}$$

$$\mathcal{A}_j = \frac{\alpha_s}{9\pi} \left[\frac{10}{9} - G(m_j, k, m_b) \right] \left[\overline{C}_2 + \frac{1}{3} \frac{\alpha}{\alpha_s} \left(3 \overline{C}_1 + \overline{C}_2 \right) \right] (1 + \chi_D), \tag{29}$$

with $j \in \{u, c\}$. The $\overline{C}_k$ refer to $\mu = m_b$ and denote the next-to-leading order scheme-independent Wilson coefficient functions introduced in Ref. [35]. The quantity

$$\chi_D = \frac{2M_D^2}{(m_c + m_d)(m_b - m_c)} \tag{30}$$

is due to the use of the equations of motion for the quark fields, whereas the function $G(m_j, k, m_b)$ originates from the one-loop penguin matrix elements of the current–current operators $Q_{1,2}^{jq}$ with internal j quarks. It is given by

$$G(m_j, k, m_b) = -4 \int_0^1 \mathrm{d}x \, x \, (1 - x) \ln \left[\frac{m_j^2 - k^2 \, x \, (1 - x)}{m_b^2} \right], \tag{31}$$

where m_j is the j-quark mass and k denotes some average four-momentum of the virtual gluons and photons appearing in the penguin diagrams [34]. In Fig. 3, we show the
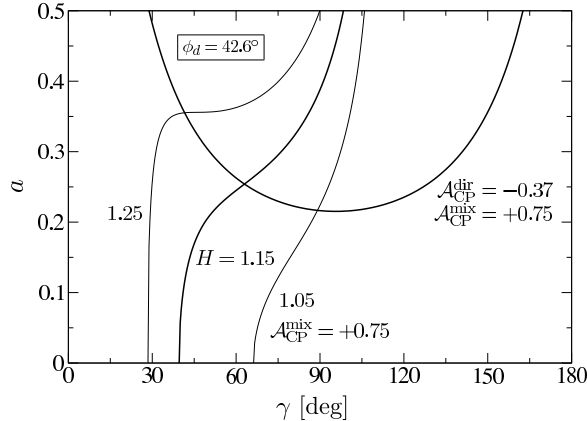
7

Figure 4: Illustration of the contours in the γ–a plane for the central values of the CP-violating $B_d \to D^+D^-$ asymmetries in (4) and various values of the ratio H of the CP-averaged $B_d \to D^+D^-$, $B_s \to D_s^+D_s^-$ branching ratios.

corresponding results, keeping k^2 as a free parameters. The sensitivity on k^2 is moderate, and in the case of H and the mixing-induced CP asymmetry even small. It should be emphasized that these results, with $a \sim 0.08$ and $\theta \sim 205°$ yielding the observables $H \sim 1.07$, $\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to D^+D^-) \sim -5\%$ and $\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to D^+D^-) \sim 76\%$, can only be considered as estimates. A similar analysis was also performed in Ref. [36]; however, in Eq. (12) of that paper, a factor of ξ is missing in front of C_3, and $10/3$ should read $10/9$.

It is instructive to compare (26) with the corresponding expression for the penguin-to-tree ratio d of the $B_d^0 \to \pi^+\pi^-$ decay in Ref. [28]. We observe that a is suppressed with respect to d by a factor of $R_b^2 \sim 0.2$. The value of $d \sim 0.4$, as determined from the U-spin analysis of the $B_d \to \pi^+\pi^-$, $B_s \to K^+K^-$ system [15], points therefore also towards $a \sim 0.08$. However, the detailed dynamics of these decays is of course very different, so that values of a at the 20% level cannot be excluded.

4 Extractions of CP-Violating Phases

4.1 Extraction of γ

As was pointed out in Ref. [2], we may combine (8) with (9) to eliminate the strong phase θ, which allows us to calculate a as a function of γ. To this end, the B_d^0–$\bar{B}_d^0$ mixing phase ϕ_d is needed as an input. The corresponding contour relies only on the SM structure of the $B_d^0 \to D^+D^-$ decay amplitude and is *theoretically clean*. A second curve of this kind can be fixed through $\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to D^+D^-)$ and H with the help of the U-spin relations in (16). The advantage of the combination of these observables is that they both depend on $\cos\theta^{(\prime)}$. Because of the ϵ suppression of the a' terms in (14), U-spin-breaking corrections to this relation have actually a very small impact, so that the major non-factorizable U-spin-breaking effects enter through the determination of H. In Fig. 4, we illustrate this strategy for the central values of the averages in (4) and different values of H. We see that $H = 1.15$ would give a value of $\gamma = 63°$ with $a = 0.25$ (and $\theta = 249°$). On the other
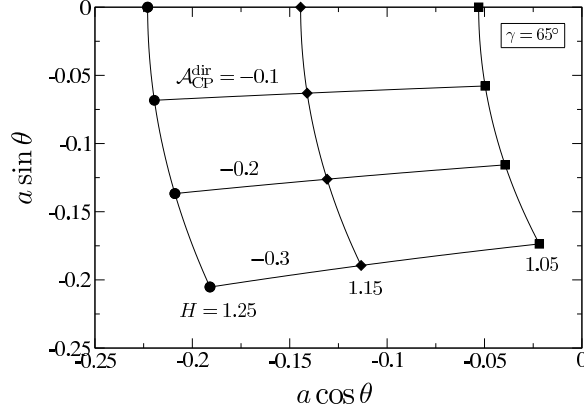
8

Figure 5: Determination of the hadronic parameter $ae^{i\theta}$ for given values of H and $\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to D^+D^-)$.

hand, $H = 1.05$ yields $\gamma = 89°$ with $a = 0.22$ and $\theta = 244°$, whereas $H = 1.25$ results in $\gamma = 42°$, $a = 0.35$ and $\theta = 257°$. Consequently, since a variation of $H = 1.15 \pm 0.10$ gives the large range of $\gamma = (63^{+26}_{-21})°$, the situation would not be favourable for the determination of this UT angle. However, the hadronic parameter $a = 0.25^{+0.10}_{-0.03}$ and – in particular the strong phase $\theta = (249^{+8}_{-5})°$ – could be well determined, but are of less interest. In the case of the U-spin-related $B_d \to \pi^+\pi^-$, $B_s \to K^+K^-$ decays, the current data result in a complementary situation, with a very favourable situation for the extraction of γ, and a less fortunate picture for the corresponding strong phase [15]. It will be interesting to follow the future evolution of the $B_{d(s)} \to D^+_{d(s)}D^-_{d(s)}$ data.

4.2 Extraction of the B_d^0–$\bar{B}_d^0$ Mixing Phase

An alternative avenue for extracting information from the CP-violating asymmetries of the $B_d^0 \to D^+D^-$ decay arises if we use γ as an input. By the time accurate measurements of these CP asymmetries will become available we will also have a clear picture of this UT angle thanks to the precision measurements that can be performed at LHCb [18]. For the following analysis, we assume a value of $\gamma = 65°$ (see the remarks after (10)).

If the penguin effects could be neglected, the following simple situation would arise:

$$(\sin 2\beta)_{D^+D^-} \equiv \mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to D^+D^-) \overset{\text{no pengs.}}{\longrightarrow} \sin\phi_d \overset{\text{SM}}{=} \sin 2\beta. \tag{32}$$

The goal of the following discussion is to include the penguin effects in the determination of $\sin\phi_d$. To this end, we first determine a through the combination of $\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to D^+D^-)$ and H by means of the U-spin relation (16), which yields

$$a = \sqrt{b - \sqrt{b^2 - c}}, \tag{33}$$

where

$$bN = 2\left[(1 + \epsilon H)\sin\gamma\cos\gamma\right]^2 + (H - 1)(1 - \epsilon^2 H)\sin^2\gamma$$

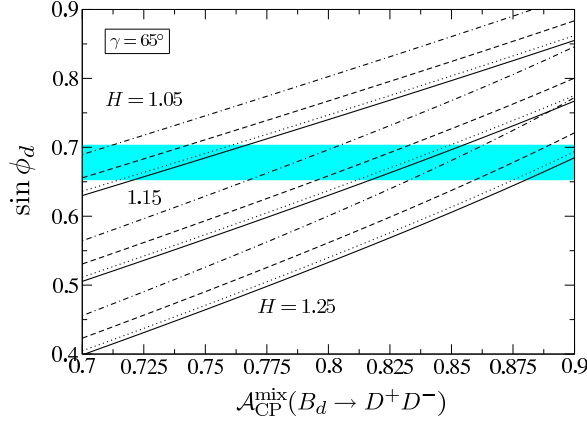$$-\epsilon\left[(1 + \epsilon)H\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to D^+D^-)\cos\gamma\right]^2 \tag{34}$$

9

Figure 6: Correlation between $\mathcal{A}_{\text{CP}}^{\text{mix}}(B_d \to D^+D^-)$ and $\sin \phi_d$ for given values of H and various values of $\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \to D^+D^-)$: 0 (solid), ± 0.1 (dotted), ± 0.2 (dashed), ± 0.3 (dot-dashed). The shaded region corresponds to the experimental value of $(\sin 2\beta)_{\psi K_{\text{S}}}$.

$$cN = \left[(H-1)\sin\gamma\right]^2 + \left[(1+\epsilon)H\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \to D^+D^-)\cos\gamma\right]^2, \tag{35}$$

with

$$N = \left[(1-\epsilon^2 H)\sin\gamma\right]^2 + \left[\epsilon(1+\epsilon)H\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \to D^+D^-)\cos\gamma\right]^2. \tag{36}$$

In (33), the sign in front of the inner square root could, in principle, be positive or negative. However, since the large values of a corresponding to the $+$ sign are completely unrealistic, we have already written the $-$ sign. The strong phase θ follows then from

$$\cos\theta = \frac{1 - H + (1 - \epsilon^2 H)a^2}{2(1+\epsilon H)a\cos\gamma} \tag{37}$$

$$\sin\theta = \left[\frac{(1+\epsilon)(1+\epsilon a^2)}{2(1+\epsilon H)a\sin\gamma}\right] H\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \to D^+D^-). \tag{38}$$

In these expressions, the impact of the ϵ terms is tiny, but we have kept them for completeness. In Fig. 5, we show the resulting picture of the hadronic parameter $ae^{i\theta}$ in the complex plane for various values of H and $\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \to D^+D^-)$, which should be compared with theoretical estimate shown in the left panel of Fig. 3.

If we now use again (16) and eliminate $\cos\theta$ in (9) through (37), we obtain

$$A\sin\phi_d + B\cos\phi_d = C, \tag{39}$$

where

$$A = \left[H - 2a^2\sin^2\gamma + \epsilon H\left\{1 + \left(1 - 2\sin^2\gamma + \epsilon\right)a^2\right\}\right]\cos\gamma \tag{40}$$

$$B = \left[H - 1 + a^2\cos 2\gamma + \epsilon H\left(1 + \cos 2\gamma + \epsilon\right)a^2\right]\sin\gamma \tag{41}$$

$$C = (1+\epsilon)(1+\epsilon a^2)H\mathcal{A}_{\text{CP}}^{\text{mix}}(B_d \to D^+D^-)\cos\gamma, \tag{42}$$

with a given in (33). Finally, $\sin\phi_d$ can be determined as follows:

$$\sin\phi_d = \frac{AC - B\sqrt{A^2 + B^2 - C^2}}{A^2 + B^2}. \tag{43}$$

10

Here we have chosen the sign in front of the square root such that we obtain a positive value of $\cos\phi_d$, in agreement with the B-factory data for the CP-violating effects in the $B_d \to J/\psi K^*$ and $B_d \to D^*D^*K_S$ channels [7]. In Fig. 6, we show the resulting correlation between the mixing-induced CP violation in $B_d^0 \to D^+D^-$ and $\sin\phi_d$ for various values of H and $\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to D^+D^-)$, which correspond to the situation shown in Fig. 5. These curves allow us straightforwardly to include the penguin effects in the determination of the B_d^0–$\bar{B}_d^0$ mixing phase form the CP-violating $B_d \to D^+D^-$ observables.

4.3 Extraction of the B_s^0–$\bar{B}_s^0$ Mixing Phase

Let us now turn to the CP-violating rate asymmetry of the $B_s^0 \to D_s^+D_s^-$ decay, which is defined in analogy to (1), and takes the form

$$
\begin{aligned}
&\mathcal{A}_{\mathrm{CP}}(B_s(t) \to D_s^+D_s^-) \\
&= \left[\frac{\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_s \to D_s^+D_s^-)\cos(\Delta M_s t) + \mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_s \to D_s^+D_s^-)\sin(\Delta M_s t)}{\cosh(\Delta\Gamma_s t/2) - \mathcal{A}_{\Delta\Gamma}(B_s \to D_s^+D_s^-)\sinh(\Delta\Gamma_s t/2)} \right],
\end{aligned} \tag{44}
$$

where $\Delta\Gamma_s \equiv \Gamma_{\mathrm{H}}^{(s)} - \Gamma_{\mathrm{L}}^{(s)}$ is the difference of the decay widths $\Gamma_{\mathrm{H}}^{(s)}$ and $\Gamma_{\mathrm{L}}^{(s)}$ of the "heavy" and "light" mass eigenstates of the B_s system, respectively. The mass difference ΔM_s was recently measured at the Tevatron [37,38], with a value that is consistent with the SM expectation. On the other hand, this result still allows for large CP-violating NP contributions to B_s^0–$\bar{B}_s^0$ mixing (see, for instance, Refs. [39,40]). In this case, the mixing phase ϕ_s would take a sizeable value, and would manifest itself also through significant mixing-induced CP violation in $B_s^0 \to D_s^+D_s^-$ at LHCb. In the SM, we have on the other hand a tiny phase of $\phi_s = -2\lambda^2\eta \approx -2°$, where η is another Wolfenstein parameter.

Using the formalism discussed in Ref. [4], (12) yields

$$
\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_s \to D_s^+D_s^-) = -\left[\frac{2\epsilon a'\sin\theta'\sin\gamma}{1 + 2\epsilon a'\cos\theta'\cos\gamma + \epsilon^2 a'^2} \right] \tag{45}
$$

$$
\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_s \to D_s^+D_s^-) = \left[\frac{\sin\phi_s + 2\epsilon a'\cos\theta'\sin(\phi_s + \gamma) + \epsilon^2 a'^2\sin(\phi_s + 2\gamma)}{1 + 2\epsilon a'\cos\theta'\cos\gamma + \epsilon^2 a'^2} \right] \tag{46}
$$

$$
\mathcal{A}_{\Delta\Gamma}(B_s \to D_s^+D_s^-) = -\left[\frac{\cos\phi_s + 2\epsilon a'\cos\theta'\cos(\phi_s + \gamma) + \epsilon^2 a'^2\cos(\phi_s + 2\gamma)}{1 + 2\epsilon a'\cos\theta'\cos\gamma + \epsilon^2 a'^2} \right], \tag{47}
$$

and (16) implies the following U-spin relation [2]:

$$
\frac{\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_s \to D_s^+D_s^-)}{\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to D^+D^-)} = -\epsilon H. \tag{48}
$$

Thanks to the suppression through the ϵ parameter in (46), the penguin effects are significantly smaller than in the case of $B_d^0 \to D^+D^-$. Nevertheless, since we are aiming at precision measurements, it is important to be able to control them. Since we may determine the penguin parameters a and θ as we have discussed above, the U-spin

relations in (16) allow us to include the penguin effects also in the determination of ϕ_s. It is instructive to perform an expansion in powers of $\epsilon a'$, which yields

$$\sin\phi_s = \mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_s \to D_s^+ D_s^-) \mp 2\epsilon a' \cos\theta' \sin\gamma \sqrt{1 - \mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_s \to D_s^+ D_s^-)^2} + \mathcal{O}((\epsilon a')^2), \tag{49}$$

where $\mp$ refers to $\mathrm{sgn}(\cos\phi_s) = \pm1$. For strategies to determine this sign, which is positive in the SM, see Refs. [15, 41]. Using (37), the relevant hadronic parameter can straightforwardly be fixed:

$$2\epsilon a' \cos\theta' \sin\gamma = (1 - H)\tan\gamma + \mathcal{O}(a^2). \tag{50}$$

Let us finally have a closer look at the observable

$$\mathcal{A}_{\Delta\Gamma}(B_s \to D_s^+ D_s^-) = -\cos\phi_s + 2\epsilon a' \cos\theta' \sin\gamma \sin\phi_s + \mathcal{O}((\epsilon a')^2), \tag{51}$$

which can be extracted from the following "untagged" rate:

$$\langle\Gamma(B_s(t) \to D_s^+ D_s^-)\rangle \equiv \Gamma(B_s^0(t) \to D_s^+ D_s^-) + \Gamma(\bar{B}_s^0(t) \to D_s^+ D_s^-)$$
$$\propto e^{-\Gamma_s t}\left[e^{+\Delta\Gamma_s t/2}R_{\mathrm{L}}(B_s \to D_s^+ D_s^-) + e^{-\Delta\Gamma_s t/2}R_{\mathrm{H}}(B_s \to D_s^+ D_s^-)\right]. \tag{52}$$

Here Γ_s denotes the average of the decay widths of the "heavy" and "light" mass eigenstates of the B_s system, and

$$R_{\mathrm{L}}(B_s \to D_s^+ D_s^-) \equiv 1 - \mathcal{A}_{\Delta\Gamma}(B_s \to D_s^+ D_s^-) = 1 + \cos\phi_s + \mathcal{O}(\epsilon a') \overset{\mathrm{SM}}{\approx} 2, \tag{53}$$

$$R_{\mathrm{H}}(B_s \to D_s^+ D_s^-) \equiv 1 + \mathcal{A}_{\Delta\Gamma}(B_s \to D_s^+ D_s^-) = 1 - \cos\phi_s + \mathcal{O}(\epsilon a') \overset{\mathrm{SM}}{\approx} 0. \tag{54}$$

As far as a practical measurement of (52) is concerned, most of the data come from short times with $\Delta\Gamma_s t \ll 1$. We may hence expand in this parameter, which yields

$$\langle\Gamma(B_s(t) \to D_s^+ D_s^-)\rangle \propto e^{-\Gamma_s t}\left[1 - \mathcal{A}_{\Delta\Gamma}(B_s \to D_s^+ D_s^-)\left(\frac{\Delta\Gamma_s t}{2}\right) + \mathcal{O}((\Delta\Gamma_s t)^2)\right]. \tag{55}$$

Moreover, if the two-exponential form of (52) is fitted to a single exponential, the corresponding decay width satisfies the following relation [41]:

$$\Gamma_{D_s^+ D_s^-} = \Gamma_s + \mathcal{A}_{\Delta\Gamma}(B_s \to D_s^+ D_s^-)\frac{\Delta\Gamma_s}{2} + \mathcal{O}((\Delta\Gamma_s)^2/\Gamma_s). \tag{56}$$

Using flavour-specific B_s decays, a similar analysis allows the extraction of Γ_s up to corrections of $\mathcal{O}((\Delta\Gamma_s/\Gamma_s)^2)$ [41]. In the presence of NP, $\Delta\Gamma_s$ is modified as follows [42]:

$$\Delta\Gamma_s = \Delta\Gamma_s^{\mathrm{SM}}\cos\phi_s, \tag{57}$$

where $\Delta\Gamma_s^{\mathrm{SM}}/\Gamma_s$ is negative for the definition given above, and calculated at the 15% level [43]. Consequently, (56) actually probes

$$\Gamma_{D_s^+ D_s^-} - \Gamma_s = \left[\cos^2\phi_s - \epsilon a' \cos\theta' \sin(2\phi_s)\right]\frac{|\Delta\Gamma_s^{\mathrm{SM}}|}{2} + \dots, \tag{58}$$

thereby complementing other determinations of the width difference of the B_s system, such as from the U-spin-related $B_s \to K^+ K^-$, $B_d \to \pi^+\pi^-$ decays [15].

5 Interplay with Other Probes of CP Violation

As we have seen in the previous section, the U-spin-related $B_q \to D_q^+ D_q^-$ decays offer an interesting tool for the extraction of the B_q^0–$\bar{B}_q$ mixing phases ($q \in \{d, s\}$). Since the "golden" decay $B_d^0 \to J/\psi K_{\rm S}$ and similar channels allow already a very impressive determination of ϕ_d, as can be seen in (10), this may not look as too exciting. However, this is actually not the case. In fact, the current value of (10) is on the lower side, and the interplay with the UT side $R_b \propto |V_{ub}/V_{cb}|$ leads to some tension in the CKM fits [7, 13, 14], which receives increasing attention in the B-physics community. If this effect is attributed to NP, the standard interpretation is through CP-violating contributions to B_d^0–$\bar{B}_d^0$ mixing, with a NP phase $\phi_d^{\rm NP} \sim -10°$ [40, 44].

However, the NP effects could also enter through the $B_d^0 \to J/\psi K_{\rm S}$ amplitude, where EW penguin topologies, which have a sizeable impact on this decay [45], offer a particularly interesting scenario. The B-factory data for $B \to \pi\pi, \pi K$ modes may actually indicate a modified EW penguin sector with a large CP-violating NP phase through the results for mixing-induced CP violation in $B_d^0 \to \pi^0 K_{\rm S}$ [16, 46], thereby complementing the pattern of such CP asymmetries observed in other $b \to s$ penguin modes, where the $B_d^0 \to \phi K_{\rm S}$ channel is an outstanding example [7]. The sign of the corresponding CP-violating NP phase would actually shift $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to J/\psi K_{\rm S})$ in the right direction [16, 45]. The interesting feature of the $B_{d(s)} \to D_{d(s)}^+ D_{d(s)}^-$ decays is that they are essentially unaffected by such a NP scenario as EW penguins contribute only in colour-suppressed form and play a minor rôle. Consequently, a difference between the values of ϕ_d extracted from $B_d \to J/\psi K_{\rm S}$ and the $B_{d(s)} \to D_{d(s)}^+ D_{d(s)}^-$ system could reveal such effects.

A similar comment applies to the determination of the B_s^0–$\bar{B}_s^0$ mixing phase, where the "golden" strategy uses mixing-induced CP violation in the time-dependent angular distribution of the $B_s \to J/\psi[\to \ell^+\ell^-]\phi[\to K^+K^-]$ decay products [41, 47]; penguin effects can be controlled with the help of $B_d \to J/\psi\rho^0$ [48]. This determination of ϕ_s could also be affected by CP-violating NP contributions entering through EW penguin topologies. On the other hand, the extraction discussed in Subsection 4.3 is essentially unaffected, so that a difference between the two results could again signal such a kind of physics beyond the SM. Moreover, also a simultaneous analysis of the U-spin-related $B_{s(d)} \to J/\psi K_{\rm S}$ decays should be performed [2]. In analogy to the discussion given above, the (small) penguin effects in the determination of ϕ_d from $B_d \to J/\psi K_{\rm S}$ can then be controlled, and ϕ_s could be extracted from the $b \to d$ channel $B_s \to J/\psi K_{\rm S}$, again with a sensitivity to a modified CP-violating EW penguin sector.

As was noted in Ref. [49], the analysis of the $B_{d(s)} \to D_{d(s)}^+ D_{d(s)}^-$ decays can also straightforwardly be applied to the $B_{d(s)} \to K^0\bar{K}^0$ system. Following these lines, the penguin effects in the determination of $\sin\phi_s$ from the $b \to s$ penguin decay $B_s^0 \to K^0\bar{K}^0$ can be included through its $B_d^0 \to K^0\bar{K}^0$ partner [50];[3] this is also the case for the corresponding $B_{d(s)} \to K^{*0}\bar{K}^{*0}$ decays [48, 51]. Again in these transitions, EW penguin have a very small impact. Should the interesting pattern in the mixing-induced CP asymmetries of $B_d^0 \to \pi^0 K_{\rm S}$, $B_d^0 \to \phi K_{\rm S}$ and similar modes originate from a modified EW penguin sector, we would again not see it in the $B_{d(s)} \to K^{(*)0}\bar{K}^{(*)0}$ system.

---

[3]Here $B_s^0 \to K^0\bar{K}^0$ and $B_d^0 \to K^0\bar{K}^0$ take the rôles of $B_s^0 \to D_s^+D_s^-$ and $B_d^0 \to D^+D^-$, respectively.

6 Conclusions

The CP violation in $B_d^0 \to D^+ D^-$ offers another interesting probe for the exploration of the Kobayashi–Maskawa mechanism of CP violation. In these studies, the penguin effects have to be controlled, which can be done with the help of the U-spin-related $B_s^0 \to D_s^+ D_s^-$ channel. Motivated by the recent data from the B factories and the Tevatron, as well as the quickly approaching start of the LHC, we have investigated the allowed region in the space of the mixing-induced and direct CP violation of the $B_d^0 \to D^+ D^-$ decay, with useful results to monitor the future improvement of the experimental picture, and have performed theoretical estimates of the relevant hadronic parameters and observables.

We then discussed the extraction of CP-violating phases, where we may either use ϕ_d as an input to determine γ, or use γ to extract ϕ_d. Concerning the former option, the current data point towards an unstable situation for the extraction of γ, while the strong phase θ could be well determined. It appears therefore more interesting to extract the B_d^0–$\bar{B}_d^0$ mixing phase from the CP asymmetries of $B_d^0 \to D^+ D^-$, also since precision measurements of γ will be available from the LHCb experiment through other strategies. We have provided the formalism to include the penguin effects, and have illustrated its practical implementation. In the case of the CP asymmetries of the $B_s^0 \to D_s^+ D_s^-$ decay, the penguin effects are doubly Cabibbo-suppressed and play therefore a significantly less pronounced rôle. However, they can also be taken into account with the help of the $B_d^0 \to D^+ D^-$ decay, allowing then a precision measurement of the B_s^0–$\bar{B}_s^0$ mixing phase from the mixing-induced CP violation in $B_s^0 \to D_s^+ D_s^-$.

An interesting feature of these determinations is the fact that they are insensitive to CP-violating NP contributions entering through the EW penguin sector. In this respect, they are complementary to the well-known standard strategies. The determinations of the B_s^0–$\bar{B}_s^0$ mixing phase through the $B_{s(d)} \to D_{s(d)}^+ D_{s(d)}^-$ system on the one hand and $B_s \to J/\psi\phi$, $B_d \to J/\psi\rho^0$ on the other hand are particularly promising, and the studies of LHCb in this direction should be further pursued to fully exploit the physics potential of these decays.

References

[1] N. Cabibbo, *Phys. Rev. Lett.* **10** (1963) 531; M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49** (1973) 652.

[2] R. Fleischer, *Eur. Phys. J.* **C10** (1999) 299.

[3] M. Gronau, J.L. Rosner and D. London, *Phys. Rev. Lett.* **73** (1994) 21; M. Gronau, O.F. Hernandez, D. London and J.L. Rosner, *Phys. Rev.* **D50** (1994) 4529.

[4] R. Fleischer, lectures given at European School of High-Energy Physics, Kitzbühel, Austria, 21 August – 3 September 2005 [hep-ph/0608010].

[5] B. Aubert *et al.* [BaBar Collaboration], arXiv:0705.1190 [hep-ex].

[6] S. Fratina *et al.* [Belle Collaboration], arXiv:hep-ex/0702031.

[7] E. Barberio *et al.* [Heavy Flavor Averaging Group (HFAG)], arXiv:0704.3575 [hep-ex]; for the most recent updates, see http://www.slac.stanford.edu/xorg/hfag/.

[8] B. Aubert *et al.* [BaBar Collaboration], *Phys. Rev.* **D73** (2006) 112004.

[9] S. Farrington *et al.* [CDF Collaboration], FERMILAB-CONF-06-405-E.

[10] K. Abe *et al.* [Belle Collaboration], arXiv:hep-ex/0610003.

[11] V.M. Abazov *et a.* [D0 Coll.], FERMILAB-PUB-07/047-E [hep-ex/0702049]

[12] L. Wolfenstein, *Phys. Rev. Lett.* **51** (1983) 1945.

[13] M. Bona *et al.* [UTfit Collaboration], *JHEP* **0507** (2005) 028; for the most recent updates, see http://utfit.roma1.infn.it/.

[14] J. Charles *et al.* [CKMfitter Group], *Eur. Phys. J.* **C41** (2005) 1; for the most recent updates, see http://ckmfitter.in2p3.fr/.

[15] R. Fleischer, CERN-PH-TH/2007-080, arXiv:0705.1121 [hep-ph].

[16] R. Fleischer, S. Recksiegel and F. Schwab, *Eur. Phys. J.* **C51** (2007) 55.

[17] M. Gronau and J.L. Rosner, arXiv:0704.3459 [hep-ph].

[18] T. Nakada, talk at CKM 2006, Nagoya, Japan, 12–16 December 2006; A. Schopper, Proceedings of FPCP 2006, Vancouver, British Columbia, Canada, 9–12 April 2006, pp 042 [hep-ex/0605113].

[19] R. Fleischer and J. Matias, *Phys. Rev.* **D66** (2002) 054009.

[20] M. Neubert and B. Stech, in *Heavy Flavours II*, eds. A.J. Buras and M. Lindner (World Scientific, Singapore, 1998), p. 294–344 [hep-ph/9705292].

[21] E. Jenkins and M.J. Savage, *Phys. Lett.* **B281** (1992) 331.

[22] B. Grinstein, E. Jenkins, A.V. Manohar, M.J. Savage and M.B. Wise, *Nucl. Phys.* **B380** (1992) 369.

[23] M. Artuso *et al.* [CLEO Collaboration], arXiv:0704.0629 [hep-ex].

[24] T. Onogi, *PoS* **LAT2006** (2006) 017.

[25] R. Fleischer, *Nucl. Phys.* **B412** (1994) 201.

[26] R. Fleischer and S. Recksiegel, *Phys. Rev.* **D71** (2005) 051501.

[27] A. Datta, talk at CKM 2006, Nagoya, Japan, 12–16 December 2006 [hep-ph/0702270]; A. Datta and D. London, *Phys. Lett.* **B584** (2004) 81.

[28] R. Fleischer, *Phys. Lett.* **B459** (1999) 306.

[29] B. Aubert *et al.* [BaBar Collaboration], *Phys. Rev.* **D74** (2006) 031103.

[30] A. Zupanc *et al.* [Belle Collaboration], *Phys. Rev.* **D75** (2007) 091102.

[31] E. Baracchini *et al.*, arXiv:hep-ph/0703258.

[32] G. Buchalla, A.J. Buras and M.E. Lautenbacher, *Rev. Mod. Phys.* **68** (1996) 1125.

[33] M. Bander, D. Silverman and A. Soni, *Phys. Rev. Lett.* **43** (1979) 242.

[34] R. Fleischer, *Z. Phys.* **C58** (1993) 483; *Z. Phys.* **C62** (1994) 81.

[35] A.J. Buras, M. Jamin, M.E. Lautenbacher and P.H. Weisz, *Nucl. Phys.* **B370** (1992) 69; A.J. Buras, M. Jamin and M.E. Lautenbacher, *Nucl. Phys.* **B408** (1993) 209.

[36] Z.-z. Xing, *Phys. Rev.* **D61** (2000) 014010.

[37] V.M. Abazov *et al.* [D0 Collaboration], *Phys. Rev. Lett.* **97** (2006) 021802.

[38] A. Abulencia *et al.* [CDF Collaboration], *Phys. Rev. Lett.* **97** (2006) 242003.

[39] M. Ciuchini and L. Silvestrini, *Phys. Rev. Lett.* **97** (2006) 021803; Z. Ligeti, M. Papucci and G. Perez, *Phys. Rev. Lett.* **97** (2006) 101801; Y. Grossman, Y. Nir and G. Raz, *Phys. Rev. Lett.* **97** (2006) 151801.

[40] P. Ball and R. Fleischer, *Eur. Phys. J.* **C48** (2006) 413.

[41] I. Dunietz, R. Fleischer and U. Nierste, *Phys. Rev.* **D63** (2001) 114015.

[42] Y. Grossman, *Phys. Lett.* **B380** (1996) 99.

[43] A. Lenz and U. Nierste, hep-ph/0612167.

[44] M. Bona *et al.* [UTfit Collaboration], *JHEP* **0603** (2006) 080; A.J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, *Eur. Phys. J.* **C45** (2006) 701.

[45] R. Fleischer, *J. Phys.* **G32** (2006) R71; *Int. J. Mod. Phys.* **A12** (1997) 2459.

[46] A.J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, *Phys. Rev. Lett.* **92** (2004) 101804; *Nucl. Phys.* **B697** (2004) 133.

[47] A.S. Dighe, I. Dunietz and R. Fleischer, *Eur. Phys. J.* **C6** (1999) 647.

[48] R. Fleischer, *Phys. Rev.* **D60** (1999) 073008.

[49] R. Fleischer, *Phys. Rept.* **370** (2002) 537.

[50] M. Ciuchini, M. Pierini and L. Silvestrini, hep-ph/0703137;

[51] S. Descotes-Genon, J. Matias and J. Virto, arXiv:0705.0477 [hep-ph].